Nonlinear Modulational Instability of Dispersive PDE Models.

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Joint with Jiayin Jin and Shasha Liao, we prove nonlinear modulational instability for both periodic and localized perturbations of periodic traveling waves for several dispersive PDEs, including the KDV type equations (e.g. Whitham equation, generalized KDV equation, Benjamin-Ono), nonlinear Schrödinger equation and BBM equation. First, the semigroup estimates required for the nonlinear proof are obtained by using the Hamiltonian structures of the linearized PDEs; Second, for KDV type equations the loss of derivative in the nonlinear term is overcome in two complementary cases. For smooth nonlinear term and general dispersive operators, we construct higher order approximation solutions and then use energy estimates to close it. For nonlinear terms of low regularity and with some additional assumption on the dispersive operators, we use a bootstrap argument to overcome the loss of derivative.